Module 3b: Flow in Pipes

Darcy-Weisbach

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Darcy-Weisbach Equation

- Based upon theory.
- Used to estimate energy loss due to friction in pipe.

$$h_f = f\frac{L}{D}\left(\frac{V^2}{2g}\right)$$

Where
h_f = head loss (feet)
L = length of pipe (feet)
f = friction factor
D = internal diameter of pipe (feet)
$V^2/2g$ = velocity head (feet)

Darcy-Weisbach Equation

- Darcy-Weisbach can be written for flow (substitute V = Q/A, where A = (π/4)D^2 in the above equation):

$$h_f = f\frac{8L}{\pi^2 D^5}\left(\frac{Q^2}{g}\right)$$

- Darcy-Weisbach can be rewritten to solve for velocity:

$$V = \left(\frac{h_f \times 2g \times D}{L \times f}\right)^{0.5}$$

Darcy-Weisbach Equation

Friction factor for Darcy-Weisbach Equation:

- Based upon the Reynolds number, N_R or Re, and a dimensionless parameter called the relative roughness, *e/d* or *ε/d* (ε = absolute roughness; d = diameter).
- For laminar flow:

$$f = \frac{64}{N_R} = \frac{64}{\mathrm{Re}}$$

- For turbulent flow, friction factor must be read off a Moody diagram (or off a relative roughness vs. friction factor diagram for completely turbulent flow).

Equivalent sand roughness diagram

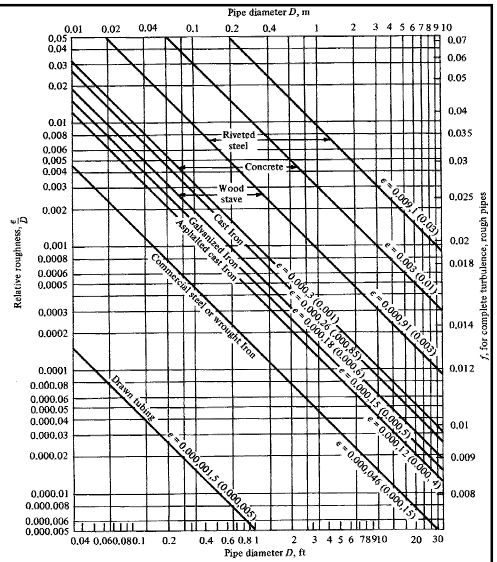
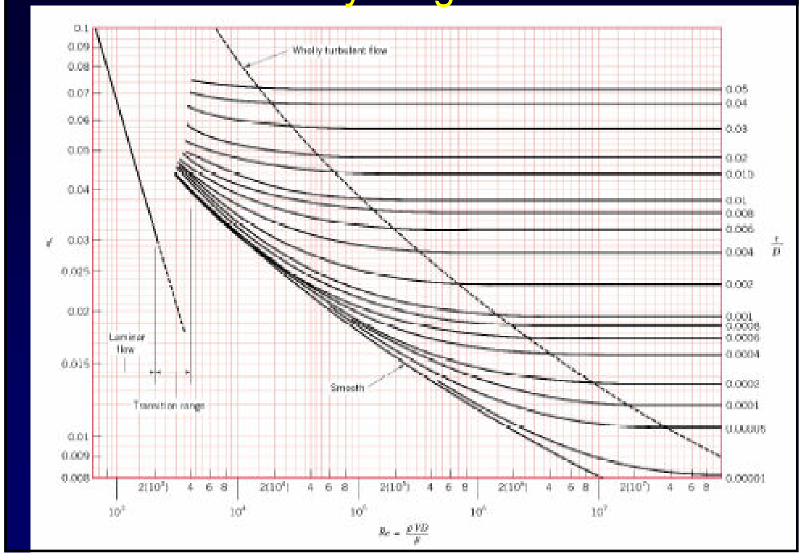


Figure 2-11 Moody diagram for relative roughness as a function of diameter for pipes constructed of various materials. Absolute roughness, ε, in m (ft.). Adapted from Ref. 13. Note: m × 3.2808 = ft.

Moody Diagram



Darcy-Weisbach Equation

- Example:
  - A polymeric coagulant, undiluted, has an absolute viscosity of 0.48 kg/(m-sec) [0.01 lb-sec/ft^2] and a specific gravity of 1.15. This fluid is to be pumped at the rate of 3.78 L/min (1 gallon/min) through 15.25 m (50 ft) of ½-inch diameter schedule 40 pipe (ID = 0.622 in = 15.8 mm = 0.0158 m). What is the head loss due to friction?
- To use Darcy-Weisbach to calculate head loss, need *f*:

$$h_f = f\frac{L}{D}\left(\frac{V^2}{2g}\right)$$

Darcy-Weisbach Equation

- If the flow is laminar, the friction factor can be calculated. Otherwise, it must be looked up off the chart.
- Need to determine velocity in order to calculate Reynolds number (determine if flow is laminar or turbulent).
- By Continuity Equation:

  Q = VA or V = Q/A
- Substituting:

$$V = \frac{[(3.78\,\mathrm{L/min})(1\,\mathrm{m}^3/1000\,\mathrm{L})(1\,\mathrm{min}/60\,\mathrm{sec})]}{[(\pi/4)(0.0158\,\mathrm{m})^2]}$$

$$V = 0.321 m/\sec$$

or

$$V = 0.32 m/\sec$$

Darcy-Weisbach Equation

- Calculate the Reynolds number to see if flow is laminar or turbulent.

$$Re = \frac{VD\rho}{\mu}$$

- Substituting:

Definition of Specific Gravity = Fluid Density/Water Density

Density of Water = ρ_{H_2O} = 1000 kg/m^3

$$Re = \frac{(0.321\text{ m/sec})(0.0158\text{ m})(1.15)(1000\text{ kg/m}^3)}{(0.48\text{ kg/m} - \text{sec})}$$

$$Re = 12.15$$

Darcy-Weisbach Equation

- HAVE LAMINAR FLOW!!
- Therefore, the friction factor for Darcy-Weisbach is calculated as follows:

$$f = \frac{64}{Re}$$

$$f = \frac{64}{12.15}$$

$$f = 5.27$$

Darcy-Weisbach Equation

- Substituting into Darcy-Weisbach equation:

$$h_f = \frac{(5.27)(15.25\text{ m})(0.321\text{ m/sec})^2}{(0.0158\text{ m})[(2)(9.806\text{ m/sec}^2)]}$$

$$h_f = 26.7m$$

- Friction Slope:

$$\text{Friction Slope} = \frac{26.7m}{15.25m} = 1.75 = 175\%$$

Darcy-Weisbach Equation

- Example:
  - A 24-inch class I ductile iron pipe (ID = 0.63 m = 24.79 in.) 90 m long with a neat cement lining (asphalted cast iron) carries a flow of water at 1.5 m^3/sec (52.9 ft^3/sec). What is the friction loss in the pipe?
  - From the relative roughness diagram (assume similar to asphalted cast iron, read ε/d.

    $\varepsilon/d = 0.0002$

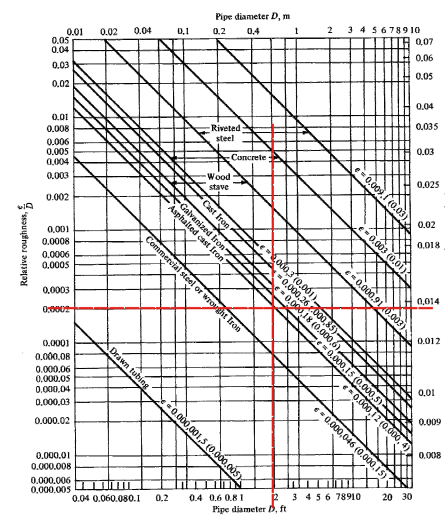


Figure 2-11 Moody diagram for relative roughness as a function of diameter for pipes constructed of various materials. Absolute roughness, ε, in m (ft.). Adapted from Ref. 13. Note: m × 3.2808 = ft.

Darcy-Weisbach Equation

- Calculate Reynolds number.

  Assuming that the fluid is water: $\nu = 1.003 \times 10^{-6}$ m²/sec at 20°C.

  – Find velocity of flow.

$$V = \frac{Q}{A} = \frac{1.5m^3/\sec}{\left[\frac{\pi}{4}(0.63m)^2\right]}$$

$$V = 4.81m/\sec$$

  – Find the Reynolds number.

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(4.81m/\sec)(0.63m)}{(1.003x10^{-6}m^2/\sec)}$$

$$\mathrm{Re} = 3.02x10^6$$

Darcy-Weisbach Equation

- Reading from the Moody Diagram:

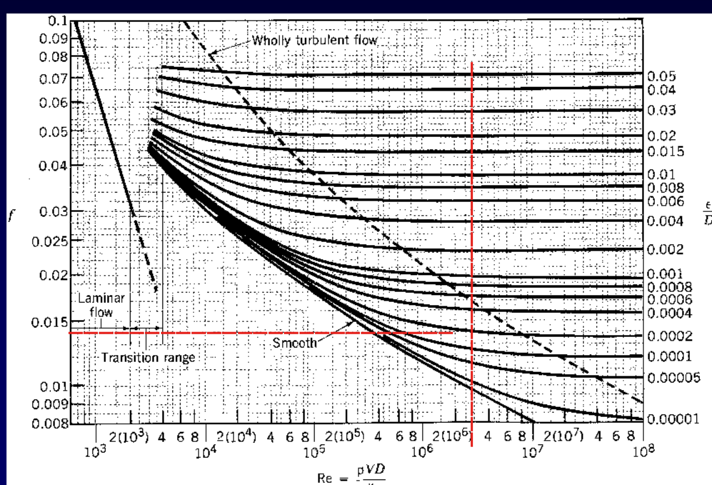
  $f(\varepsilon/d = 0.0002$ & $\mathrm{Re} = 3.02 \times 10^6) = 0.014$

- Substituting into Darcy-Weisbach:

$$h_f = \frac{(0.014)(90\ \mathrm{m})(4.81\ \mathrm{m/sec})^2}{(0.63\ \mathrm{m})(2)(9.806\ \mathrm{m/sec^2})}$$

$$h_f = 2.36m$$

Moody Diagram



Darcy-Weisbach Equation

- Example:
- Determine the flowrate in a 500-m section of a 1-m diameter commercial steel pipe when there is a 2-m drop in the energy grade line over the section.

  Given: L = 500 m

  D = 1 m

  Commercial steel pipe

  $h_f = 2$ m

- Want to use Darcy-Weisbach equation with flow rate.

$$h_f = f\frac{8L}{\pi^2 D^5}\left(\frac{Q^2}{g}\right)$$

Darcy-Weisbach Equation

- Need to find the friction factor.
  - As a first assumption about the flow in the pipe, will assume fully turbulent flow. Using Moody diagram for relative roughness in turbulent flow:

$f = 0.0105$

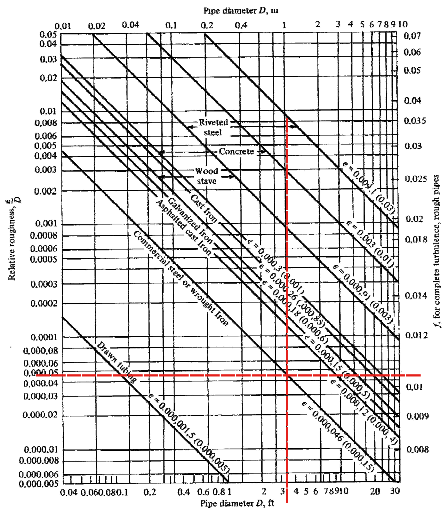


Figure 2-11 Moody diagram for relative roughness as a function of diameter for pipes constructed of various materials. Absolute roughness, ε, in m (ft.). Adapted from Ref. 13. Note: m × 3.2808 = ft.

Darcy-Weisbach Equation

- Solve Darcy-Weisbach for flow rate:

$$h_f = f\frac{8L}{\pi^2 D^5}\left(\frac{Q^2}{g}\right)$$

$$Q^2 = \frac{h_f \pi^2 g D^5}{8fL}$$

$$Q = \left[\frac{h_f \pi^2 g D^5}{8fL}\right]^{1/2}$$

Substituting:

$$Q = \left[\frac{(2m)\pi^2(9.81m/\sec^2)(1m)^5}{8(0.0105)(500m)}\right]^{1/2}$$

$$Q = 2.1472 m^3/\sec$$

Darcy-Weisbach Equation

- Must check assumption of fully turbulent flow (i.e., was the *f* selected from the figure acceptable?).
- Using the continuity equation:

$$Q = VA = \frac{\pi}{4}VD^2$$

$$2.1472m^3/\sec = \frac{\pi}{4}V(1.0m)^2$$

$$V = 2.7339 m/\sec$$

Darcy-Weisbach Equation

- Calculate the Reynolds number. (Note that no temperature is given for the fluid. Assume the fluid is water and the temperature is 15°C).

  At 15°C, $\nu = 1.139 \times 10^{-6}$ m²/sec
- Substituting:

$$\mathrm{Re} = \frac{VD}{\nu}$$

$$\mathrm{Re} = \frac{(2.7339\,m/\sec)(1.0m)}{1.139x10^{-6}m^2/\sec}$$

$$\mathrm{Re} = 2.40x10^6$$

Darcy-Weisbach Equation

- In order to use the full Moody diagram, need the relative roughness. For commercial steel, ε = 0.000046 m.
- Calculate:

ε/D = (0.000046 m)/ (1.0 m) = 0.000046

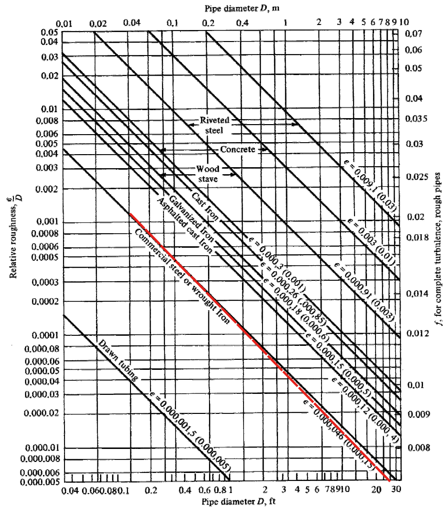
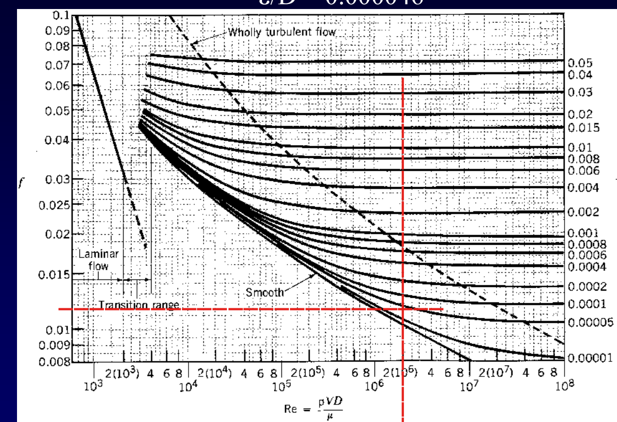


Figure 2-11 Moody diagram for relative roughness as a function of diameter for pipes constructed of various materials. Absolute roughness, ε, in m (ft.). Adapted from Ref. 13. Note: m × 3.2808 = ft.

Moody Diagram: Draw imaginary line interpolating at ε/D = 0.000046



Using the full Moody diagram, for Re = 2.40 x 10^6 and ε/D = 0.000046: $f = 0.0115$

Darcy-Weisbach Equation

- Substituting back into Darcy-Weisbach for flow:

$$Q = \left[\frac{h_f \pi^2 g D^5}{8 f L}\right]^{1/2}$$

Substituting:

$$Q = \left[\frac{(2m)\pi^2 (9.81 m/\sec^2)(1m)^5}{8(0.0115)(500m)}\right]^{1/2}$$

$$Q = 2.05 m^3/\sec$$

Are we done?

Darcy-Weisbach Equation

- To check this flow rate, repeat the process.
- By continuity:

$$V = \frac{Q}{A} = \frac{2.05 m^3/\sec}{\left(\frac{\pi}{4}\right)(1.0m)^2}$$

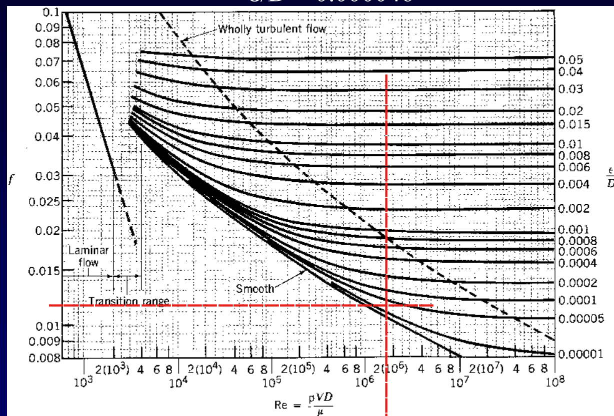
$$V = 2.61 m/\sec$$

- Calculate the Reynolds number associated with this velocity:

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(2.61 m/\sec)(1.0m)}{1.139x10^{-6} m^2/\sec}$$

$$\mathrm{Re} = 2.3x10^6$$

Moody Diagram: Draw imaginary line interpolating at $\varepsilon/D = 0.000046$



Using the full Moody diagram, for $Re = 2.30 \times 10^6$ and $\varepsilon/D = 0.000046$: $f \approx 0.0115$

Darcy-Weisbach Equation

- Friction factors approximately the same between last two iterations. Can use value from previous iteration as Q.

$Q = 2.05\ m^3/sec$

Friction factors approximately the same between last two iterations.

Are we done? Yes.

Darcy-Weisbach Equation

Example:

- A 14-inch schedule 80 pipe (commercial steel) has an inside diameter of 12.5 in (317.5 mm). How much flow can this pipe carry if the allowable head loss is 3.5 m in a length of 200 m?
- Need to solve for V and A to get Q, the flow.

Darcy-Weisbach Equation

- In order to use the full Moody diagram, need the relative roughness. For commercial steel, $\varepsilon = 0.000046$ m.
- Calculate:

$\varepsilon/D = 0.0046$ mm/ 317.5 mm = 0.000014

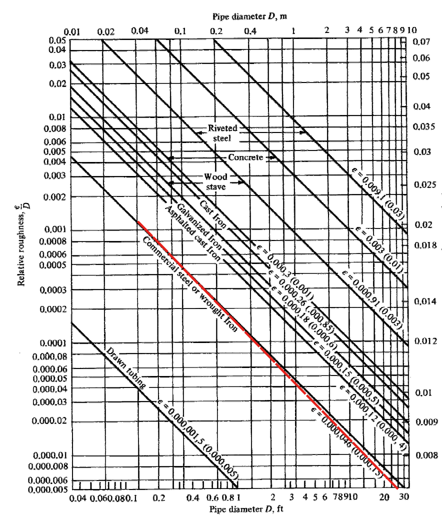
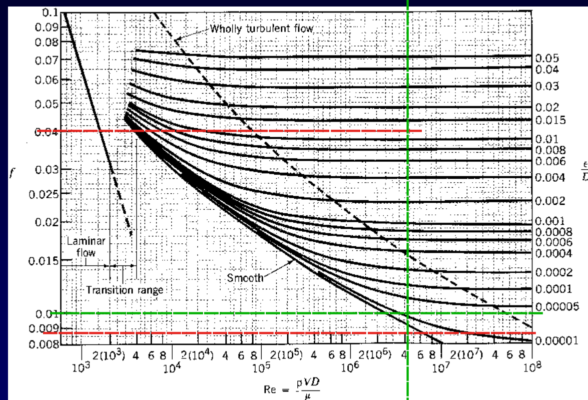


Figure 2-11 Moody diagram for relative roughness as a function of diameter for pipes constructed of various materials. Absolute roughness, ε, in m (ft.). Adapted from Ref. 13. Note: m × 3.2808 = ft.

Refer to Moody Diagram



For this ε/d, f ranges from 0.0087 to 0.04, depending on the flow (expressed as the Reynolds number).
Assume $f = 0.01$ (in range near low end). If $f = 0.01$, then the Reynolds number is approximately 4 x 10^6.

Darcy-Weisbach Equation

- Substituting this into Darcy-Weisbach:

$$V = \frac{[h_f \times 2g \times D]^{0.5}}{[f \times L]^{0.5}}$$

$$V = \frac{[(3.5\text{ m})(2)(9.806\text{ m/sec}^2)(0.3175\text{ m})]^{0.5}}{[(200\text{ m})(0.01)]^{0.5}}$$

$$V = 3.30 m/\text{sec}$$

Are we done?

Darcy-Weisbach Equation

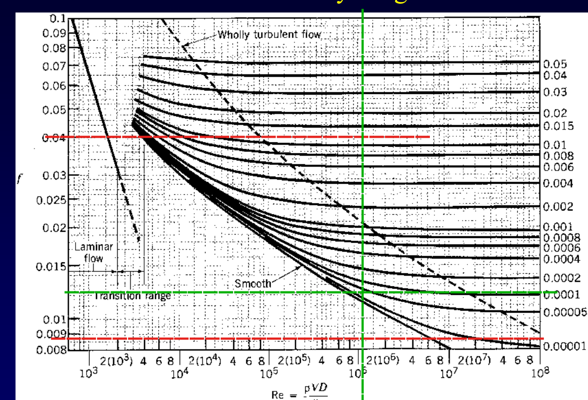
- Now need to check the friction factor assumption:
  - Calculate Reynolds number for V = 3.30 m/sec.

$$\text{Re} = \frac{VD}{\nu}$$

$$\text{Re} = \frac{3.30 m/\text{sec}(0.3175 m)}{1.003x10^{-6} m^2/\text{sec}}$$

$$\text{Re} = 1.04x10^6$$

Refer to Moody Diagram



From the Moody diagram, the f for this Reynolds number is 0.012.

Darcy-Weisbach Equation

- Recalculate using Darcy-Weisbach:

$$V = \frac{[h_f \times 2g \times D]^{0.5}}{[f \times L]^{0.5}}$$

$$V = \frac{[(3.5\,\text{m})(2)(9.806\,\text{m/sec}^2)(0.3175\,\text{m})]^{0.5}}{[(200\,\text{m})(0.012)]^{0.5}}$$

$$V = 3.01 m/\text{sec}$$

Darcy-Weisbach Equation

- Calculate the Reynolds number for V = 3.01 m/sec.

$$\text{Re} = \frac{VD}{\nu}$$

$$\text{Re} = \frac{(3.01\,\text{m/sec})(0.3175\,\text{m})}{1.003x10^{-6}\,\text{m}^2/\text{sec}}$$

$$\text{Re} = 0.95x10^6$$

CLOSE ENOUGH, So pipe will carry:

$$Q = VA = (3.01 m/\text{sec})\left(\frac{\pi}{4}\right)(0.3175 m)^2$$

$$Q = 0.238 m^3/\text{sec}$$

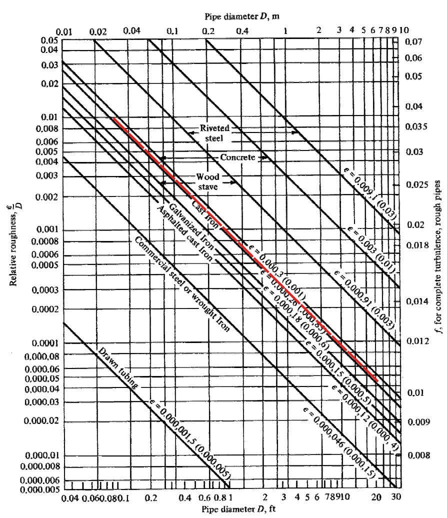
Are we done? Yes.

Darcy-Weisbach Equation

Example:

- Determine the head loss in a 46-cm concrete pipe with an average velocity of 1.0 m/sec and a length of 30 m.

- Calculate ε/d.
  D = 46 cm = 0.46 m
  ε = 0.001 ft (0.3048 m/ft) = 0.000305 mm
  Therefore ε/d = 0.0003 m/0.46 m = 0.00066



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Darcy-Weisbach Equation

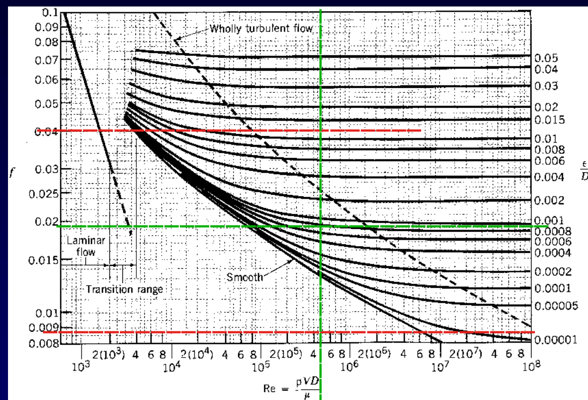
- Calculate Reynolds number:

$$\text{Re} = \frac{VD}{\nu}$$

$$\text{Re} = \frac{(1\,\text{m/sec})(0.46\,\text{m})}{1.003x10^{-6}\,\text{m}^2/\text{sec}}$$

$$\text{Re} = 4.58x10^5$$

Refer to Moody Diagram



From the Moody diagram, the *f* for this Reynolds number is 0.019.

Darcy-Weisbach Equation

- Substituting into Darcy-Weisbach:

$$h_L = f\frac{L}{D}\left(\frac{V^2}{2g}\right)$$

$$h_L = (0.019)\left(\frac{30m}{0.46m}\right)\left(\frac{(1m/\sec)^2}{2(9.806m/\sec^2)}\right)$$

$$h_L = 0.063m$$

Are we done? Yes.